

# Section Notes #1

Agenda:

- ① Hello, section time / misc
- ② An absorptive atmosphere
- ③ A multi-layer atmosphere
- ④ An abyssal temperature profile

So we know:

$$T_S^4 = T_n^4 + T_1^4$$

$$T_1^4 - T_n^4 = (5-k) [T_2^4 - T_3^4]$$

$$\Rightarrow T_1^4 - T_S^4 = T_n^4$$

Then since  $T_n^4 - T_S^4 = n \cdot (-T_n^4)$

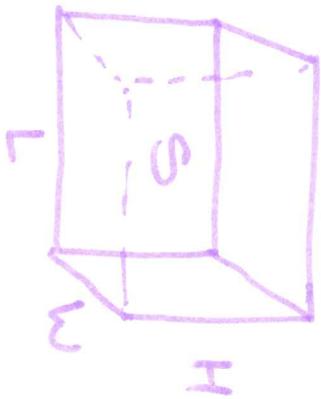
$$\Rightarrow T_S^4 = (n+1)T_n^4 = (n+1) \frac{S_0}{4} (1-\alpha)$$

$$T_S = (n+1)^{1/4} \left( \frac{S_0}{4} (1-\alpha) \right)^{1/4} = (n+1)^{1/4} T_0!$$



(5)

A simple way to think about time rates of change



I have an amount,  $\$$ , of "stuff" per square cubic meter. The total amount of stuff is then  $S \cdot W \cdot H \cdot L$

Suppose I have an amount of stuff that is passing from left to right, with speed  $V$

Then at the left edge,  $V \cdot S(x - \frac{1}{2}) \cdot W \cdot H$  comes in, and at this right edge,  $V \cdot S(x + \frac{1}{2}) \cdot W \cdot H$  leaves



More generally, imagine  $S_i$  moves in from the left per second, so the total additional stuff is  $\dot{S}_0 \cdot w \cdot H \cdot \Delta t \Big|_{x-L/2} - \dot{S}_0 \cdot w \cdot H \cdot \Delta t \Big|_{x+L/2}$

The time rate of change is then

$$\frac{\Delta S}{\Delta t} = \frac{\dot{S}_0(x-L/2) - \dot{S}_0(x+L/2)}{L}$$

$$\Rightarrow \frac{\partial S}{\partial t} = -\frac{\partial \dot{S}_0}{\partial x}$$

For advection,  $\dot{S}_0 = w \cdot T$   
 " diffusion,  $\dot{S}_0 = -\kappa \frac{\partial T}{\partial x}$

$$\Rightarrow \frac{\partial \Phi}{\partial t} = -w \cdot \frac{\partial T}{\partial x} + \kappa \frac{\partial^2 T}{\partial x^2}!$$



Suppose  $u_T z = kTz^2$

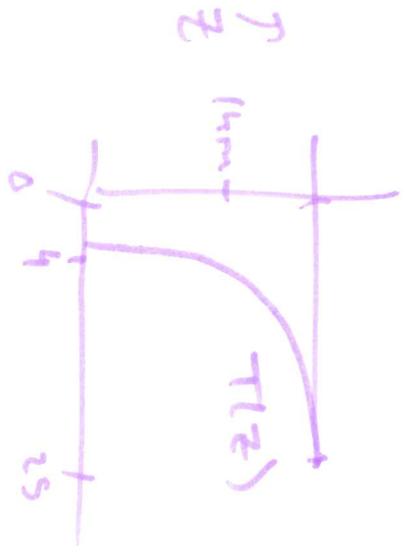
$$T(1 \text{ km}) = 4^\circ$$

$$T(0) = 25$$

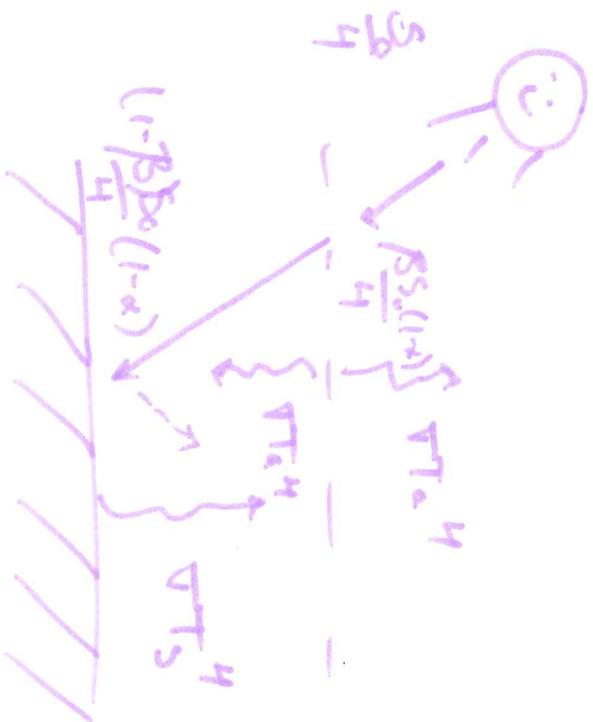
then  $T = C + D e^{-\frac{k}{3}z}$

$$\Rightarrow C + D = 25$$

$$4 = C + D e^{-\frac{k}{3}(1 \text{ km})} \quad \left. \vphantom{4 = C + D e^{-\frac{k}{3}(1 \text{ km})}} \right\} \text{ solve!}$$



② What if some radiation is absorbed in the atmosphere?



Then

$$2\sigma T_a^4 = \frac{\beta S_0}{4} (1-\alpha) + \sigma T_s^4$$

$$\sigma T_s^4 = \frac{(1-\beta) S_0}{4} (1-\alpha) + \sigma T_a^4$$

$$\Rightarrow \sigma T_s^4 = \frac{(1-\beta) S_0}{4} (1-\alpha) + \frac{\beta S_0}{8} (1-\alpha) + \frac{\sigma}{2} T_s^4$$

$$S_0 \sim 1367 \quad \sigma T_s^4 \left[ 1 - \frac{1}{2} \right] = \frac{S_0}{4} (1-\alpha) \left[ 1 - \beta + \beta/2 \right]$$

$$\alpha = .25$$

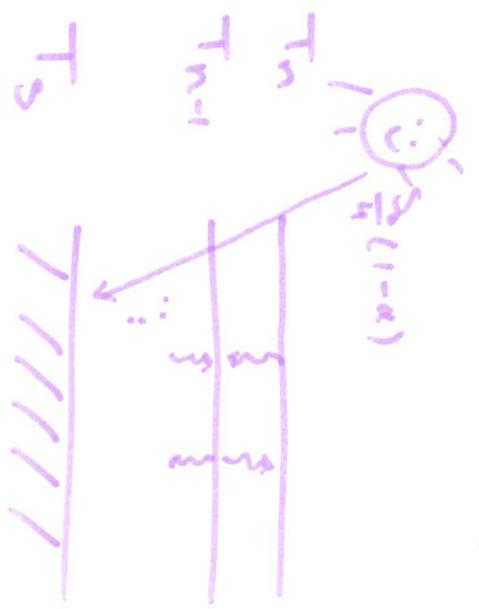
$$\beta = .1$$

$$T_s = \left[ \frac{S_0}{2\sigma} (1-\alpha) (1-\beta/2) \right]^{1/4} \approx 304^\circ \text{K}$$

Hot!

②

③ Multi-Layer Atmosphere:



Then

$$\frac{S_0}{4}(1-\alpha) + \sigma T_1^4 = \sigma T_s^4$$

and

$$\sigma T_{n+1}^4 + \sigma T_{n-1}^4 = 2\sigma T_n^4$$



subtract  $\sigma T_n^4$  from both sides  
and rearrange...

$$\sigma [T_{n+1}^4 - T_n^4] = \sigma [T_n^4 - T_{n-1}^4]$$

or

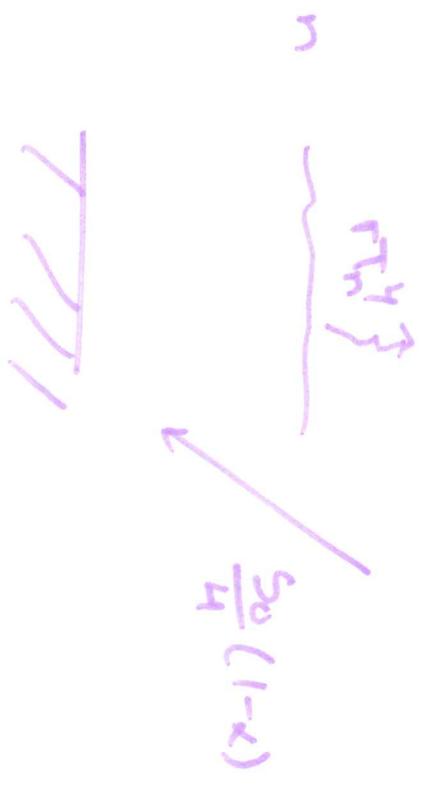
$$\sigma \Delta T_{n+1}^4 = \sigma \Delta T_n^4$$



What about two arbitrary indices? ( $j > k$ )

$$\begin{aligned}
 T_j^4 - T_k^4 &= (T_j^4 - T_{j-1}^4) + (T_{j-1}^4 - T_{j-2}^4) + \dots + T_{k+2}^4 - T_{k+1}^4 + T_k^4 \\
 &= (T_1^4 - T_2^4) + (T_2^4 - T_3^4) + \dots + (T_k^4 - T_{k+1}^4) + T_k^4 \\
 &= (j-k)(T_1^4 - T_j^4)
 \end{aligned}$$

What can we say about the entire atmosphere?



well,  $\nabla T_n^4 = \frac{S_0}{4} (1-\alpha)$  !

but  $\nabla T_j^4 = \frac{S_0}{4} (1-\alpha) + \nabla T_1^4$

$\Rightarrow \nabla T_j^4 = \nabla T_n^4 + \nabla T_1^4$  !