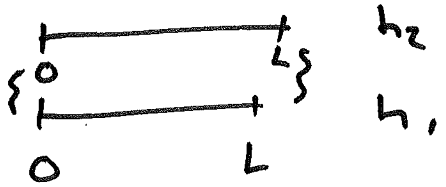
② Unforced Delay Eqn.



$$\left(\frac{\partial}{\partial t} + c_1 \frac{\partial}{\partial x}\right) h_1 = -\tau h_1 \qquad (1)$$

$$\left(\frac{\partial}{\partial t} + c_2 \frac{\partial}{\partial x}\right) h_2 = -\tau h_2 \qquad (2)$$

$$h_1(0, t - L/c_1) = \overset{\circ}{h}_1 \qquad h_1(0,t) = \gamma h_2(0,t)$$

$$h_2(L, t - L/c_2) = \overset{\circ}{h}_2 \qquad h_1(L,t) = \beta h_2(L,t)$$

Let's solve:

Let $(\eta, s) \equiv (x - c_1 t, t)$

then $\frac{\partial h}{\partial s} = \frac{\partial h}{\partial t}\frac{\partial t}{\partial s} + \frac{\partial h}{\partial x}\frac{\partial x}{\partial s}$

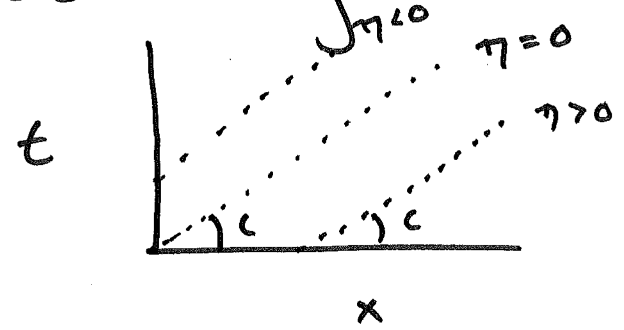
Fixing $\eta \Rightarrow x = \eta + c_1 t = \eta + c_1 s$

$$\rightarrow \frac{\partial x}{\partial s} = c_1$$

$$\frac{\partial t}{\partial s} = 1$$

$$\Rightarrow \frac{\partial h_1}{\partial s} = \frac{\partial h_1}{\partial t} + c_1 \frac{\partial h_1}{\partial x} = -\tau h_1$$

So along a contour of fixed η



$\eta = x - ct$

$$\frac{dh_1(\eta,s)}{ds} = -\tau$$

$$\Rightarrow h_1(\eta,s) = c(\eta)e^{-\tau s}$$

what about for (L,t) ?

then $\eta = L - ct \equiv \eta^*$, $s = t$

$$h_1(\eta^*, s) = c(\eta^*)e^{-\tau s}$$

what is our b.c. ?

$$h_1(0, t - L/c) = h_1^0$$

there $\eta = L - ct = \eta^*$! $\quad s = t - L/c$

so $h_1(\eta^*, t - L/c) = c(\eta^*)e^{-\tau(t - L/c)} = h_1^0$

$$\Rightarrow c(\eta^*) = h_1^0 e^{\tau(t - L/c)}$$

$$\Rightarrow h_1(L,t) = h_1^0 e^{\tau(t-L/c)} e^{-\tau t} = h_1^0 e^{-\tau L/c}$$

Let us not be discouraged, as Letting $\Delta_1 \equiv L/c_1$,

$$h_1(L,t) = h_1^0 e^{-\tau\Delta}$$

$$= h(0, t-\Delta_1) e^{-\tau\Delta_1}$$

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For h_2, the same is true:

$$\Delta_2 \equiv L/c_2$$

$$h_2(0,t) = h_2^0 e^{-\tau\Delta_2}$$

$$= h_2(L, t-\Delta_2) e^{-\tau\Delta_2}$$

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Now $h_1(0,t) = \gamma h_2(0,t)$

$$\Rightarrow h_1(L,t) = \gamma h_2(0, t-\Delta_1) e^{-\tau\Delta_1}$$

$$= \gamma h_2(L, t-\Delta_1-\Delta_2) e^{-\tau\Delta_2 - \tau\Delta_1}$$

and $h_1(0,t) = \beta h_2(L,t)$

$$\Rightarrow h_1(L,t) = \gamma\beta h_1(L, t-\Delta_1-\Delta_2) e^{-\tau(\Delta_1+\Delta_2)}$$

$\square$