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"Mini - Section"

1. - Solving for equatorial wave soln's
2. - Solving an unforced delay eqn.
3. - Rossby wave ray tracing for simple U.

! Pick one !

1. Recall our eqn.s

(1) $U_t - \beta y v = -g' \dfrac{\partial h}{\partial x}$ ← zonal mom.

(2) $V_t + \beta y u = -g' \dfrac{\partial h}{\partial y}$ ← meridional mom.

(3) $\dfrac{\partial h}{\partial t} + H(U_x + V_y) = 0$ ← cons. of mass

Lets look for wave solns that travel @ the equator

- $u(x,y,t) = \hat{u}(y) e^{i(kx - \omega t)}$
- $v(x,y,t) = \hat{v}(y) e^{i(kx - \omega t)}$
- $h(x,y,t) = \hat{h}(y) e^{i(kx - \omega t)}$

w/

$$\frac{\partial u}{\partial x} = ik\hat{u} e^{()}$$

$$\frac{\partial u}{\partial y} = \frac{\partial \hat{u}}{\partial y} e^{()}$$

$$\frac{\partial u}{\partial t} = -i\omega \hat{u} e^{()}$$

etc, etc..

Then (1) - (3) become ... (dividing by $e^{()} \neq 0$)

$$-i\omega\hat{u} - \beta y \hat{v} = -ikg'\hat{h} \quad (4)$$

$$-i\omega\hat{v} + \beta y \hat{u} = -g'\frac{\partial \hat{h}}{\partial y} \quad (5)$$

$$-i\omega\hat{h} + H\left(ik\hat{u} + \frac{\partial \hat{v}}{\partial y}\right) = 0 \quad (6)$$

From (4),

$$\hat{u} = \frac{kg'\hat{h}}{\omega} + \frac{i\beta y}{\omega}\hat{v}$$

$$\Rightarrow \hat{v}\left[\frac{i\beta y}{\omega} - \frac{i\omega}{\beta y}\right] = -\frac{g'}{\beta y}\left[\frac{\partial \hat{h}}{\partial y} + \frac{kh}{\omega}\right]$$

From (5)

$$\hat{u} = \frac{-g'}{\beta y}\frac{\partial \hat{h}}{\partial y} + \frac{i\omega\hat{v}}{\beta y} \longrightarrow \hat{v}\left[\beta^2 y^2 - \omega^2\right] = \frac{i\omega g'}{\beta y}\left[h_y + \frac{kh}{\omega}\right]$$

From (6)

$$\hat{u} = \frac{i}{k}\frac{\partial \hat{v}}{\partial y} + \frac{\omega}{Hk}\hat{h} \longrightarrow \hat{h}\left[\frac{\omega}{Hk} - \frac{kg'}{\omega}\right] = \frac{i\beta y}{\omega}\hat{v} - \frac{i}{k}\frac{\partial \hat{v}}{\partial y}$$

$$\longrightarrow \hat{h}\left[\omega^2 - g'Hk^2\right] = i\omega H\left[-\frac{\partial \hat{v}}{\partial y} + \frac{k\beta y}{\omega}\hat{v}\right]$$

Taking $\frac{\partial}{\partial y}(*)$, $\frac{\partial}{\partial y}(\square)$

$$\longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y}\right) + \left(\frac{\omega^2}{g'H} - k^2 - \frac{k\beta}{\omega} - \frac{\beta^2}{g'H}y^2\right)\hat{v} = 0$$

which admits parabolic solns w/

$$\frac{\omega^2}{c^2} - k^2 - \frac{\beta k}{\omega} = (2n+1)\frac{\beta}{c}$$

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