③ Rossby wave tracing

Let $\bar{U}_{M}(\phi) = a\omega\cos\phi + Ba\ln\left[\frac{(1+\sin\phi)}{\cos\phi}\right]\cos\phi$

then $\bar{U}_M(\phi) = a\omega + \frac{Ba}{\cancel{\cos\phi}}\ln(\ )\cancel{\cos\phi}$

$= \bar{U}_{M,\text{fixed}} + \bar{U}_{M,\text{spec.}}$

For β_M: we know $\beta_{M,\text{fixed}} = \frac{2\Omega}{a}\cos^2\phi + \frac{2\omega}{a}\cos^2\phi$

then $\beta_{M,\text{spec}} = -\frac{\partial}{\partial y}\left[\frac{1}{\cos^2\phi}\frac{\partial}{\partial y}\left(\frac{Ba\ln(\ )}{\cos^2\phi}\right)\right]$

but $y = a\ln\left(\frac{(1+\sin\phi)}{\cos\phi}\right)$

$\Rightarrow \beta_{MS} = -B\frac{\partial}{\partial y}\left[\frac{1}{\operatorname{sech}^2(y/a)}\frac{\partial}{\partial y}\left(\operatorname{sech}^2(y/a)\, y\right)\right]$

$= -B\frac{\partial}{\partial y}(y) - \frac{B}{a}\frac{\partial}{\partial y}\left[-2\tanh(y/a)\,y\right]$

$= 0 + \frac{2B}{a^2}\left[\operatorname{sech}^2(y/a)y + a\tanh(y/a)\right]$

$= \frac{2B}{a^2}\left[\cos^2\phi\, y + a\sin\phi\right]$

$$\beta_{M,s} = \frac{2B}{a^2}\left[\cos^2\phi\, y + a\sin\phi\right]$$

Earth constant effect wind

$$\Rightarrow \beta_M = \frac{2\cos^2\phi}{a}\left[\Omega + \omega + \frac{By}{a}\right] + \frac{2B\sin\phi}{a}$$

w/ $y = a\ln\left(\frac{(1+\sin\phi)}{\cos\phi}\right)$

$$k_s = \sqrt{\frac{\beta_M}{\bar{U}_M}} = \frac{(\quad)^{1/2}}{(\quad)^{1/2}}, \ldots$$

when

$\omega = \bar{U}_M k - \frac{\beta_M k}{k^2+l^2}$ RW disp. rel.

remembering $k^2 + l^2 = k_s^2$ for $\omega = 0$

$$\Rightarrow \text{w/ } \frac{dy}{dx} = \frac{v_g}{u_g} = \frac{2\beta_M k l}{(k^2+l^2)^2} \frac{(k^2+l^2)^2}{2\beta_M k^2} = \frac{l}{k}$$

$$l = \sqrt{k_s^2 - k^2}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{k_s^2}{k^2} - 1}$$