Section 4 — 2/25/14

1. HW ?'s ??
2. Inertial Oscillations and e^{ikx}
3. Maybe: 2-D wave-vectors + phase velocity

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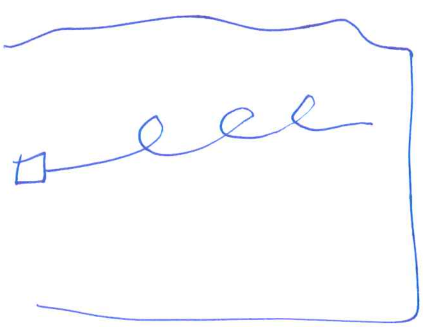
2. Inertial Oscillations

Remember:

$$\frac{\partial u}{\partial t} = fv$$

$$\frac{\partial v}{\partial t} = -fu$$



A balance between the Coriolis Force + acceleration

$$\frac{\partial u}{\partial t} = fv \qquad \Rightarrow \qquad \underbrace{\frac{\partial}{\partial t}\left(-\frac{1}{f}\frac{\partial v}{\partial t}\right)}_{} = fu$$

$$\frac{\partial v}{\partial t} = -fu$$

$$= -\frac{1}{f}\frac{\partial^2 v}{\partial t^2} = fu$$

$\Rightarrow v_{tt} = -f^2 v$. Linear + Homogenous

So we know $v(t) \sim Ae^{kt}$.

Plugging in, $\cancel{A}k^2 \cancel{e^{kt}} = -F^2 \cancel{A}\cancel{e^{kt}}$

$$\Rightarrow k^2 = -f^2 \longrightarrow k = \pm i F$$

ω

$i = \sqrt{-1}$

then $v(t) = A \cdot e^{iF} + Be^{-iF}$.

$\boxed{\text{Brief Mathematical Diversion}}$

The Taylor/Maclaurin series for $\cos(x)$ is $\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \dots \sum_{k \text{ even}} (-1)^{\frac{k}{2}} \frac{x^k}{k!}$

" " " " $\sin(x)$ is $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + \sum_{k \text{ odd}} (-1)^{\frac{k-1}{2}} \frac{x^k}{k!}$

" " " " e^x is $e^x = 1 + x + \frac{x^2}{2!} + \dots + \sum_k \frac{x^k}{k!}$

So then e^{ix} is $e^{ix} = 1 + ix - \frac{x^2}{2!} - i\frac{x^3}{3!} + \frac{x^4}{4!}$

$= \cos(x) + i\sin(x)$!

Thanks, Euler!

So if $v(t) = Ae^{ift} + Be^{-ift}$

$$\Rightarrow v(t) = A[\cos(ft) + i\sin(ft)] + B[\cos(-ft) + i\sin(-ft)]$$

<u>Remembering</u> (or just look back at our BMD last page)

$\cos(x) = \cos(-x)$
$\sin(x) = -\sin(-x)$

$$\Rightarrow v(t) = [A+B]\cos(ft) + i[A-B]\sin(ft)$$
$$= A^*\cos(ft) + B^*\sin(ft)$$

where A^*, B^* are complex numbers.

<u>Then</u> if $v(0) = V_0$ $u(0) = 0$

Remember $V_t = -fu \Rightarrow -fu = A^* f\sin(ft) - fB^*\cos(ft)$

$$\Rightarrow u = -B^*\cos(ft) + A^*\sin(ft)$$

$v(0) = A^* \cdot 1 + B^* \cdot 0 = V_0 \Rightarrow A^* = V_0$
$u(0) = -B^* \cdot 1 + A^* \cdot 0 = 0 \Rightarrow B^* = 0$

$$\Rightarrow \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} V_0\sin(ft) \\ V_0\cos(ft) \end{bmatrix}$$

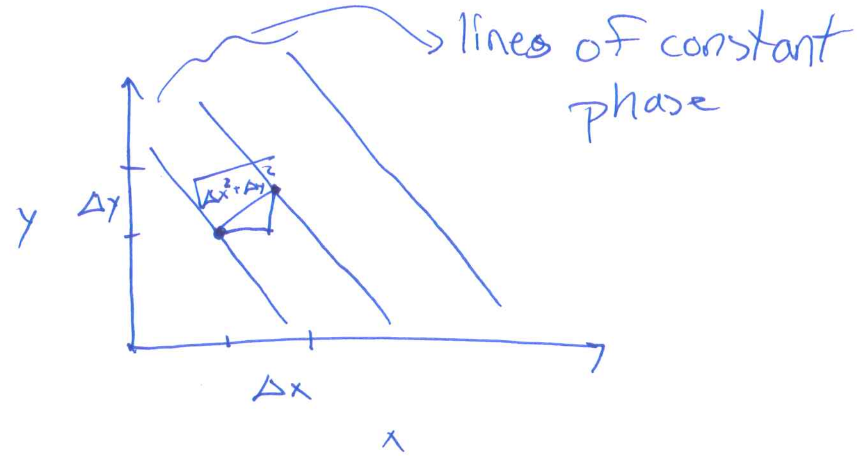
and $$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \int_0^t \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} dt = \begin{bmatrix} X_0 + \frac{V_0}{f}\cos(ft) \\ Y_0 - \frac{V_0}{f}\sin(ft) \end{bmatrix}$$

2-D wave + wave-vector

Consider the wave

$$\eta(x,y,t) = \eta_0 \cos(kx + ly - \omega t)$$

what is its phase velocity in the direction of propagation?



Consider the wavevector $\underset{\sim}{K} = (k, l)$

Then $\eta(\underset{\sim}{x}, t) = \eta_0 \cos(\underset{\sim}{K} \cdot \underset{\sim}{x} - \omega t)$ when $\underset{\sim}{x} = (x,y)$

$\eta(0,0) = \eta_0 = \eta_0 \cos(\underset{\sim}{k} \cdot \underset{\sim}{x} - \omega t)$ → along the direction of propagation

when $\underset{\sim}{k} \cdot \underset{\sim}{x} - \omega t = 0 + 2\pi i \quad i \in \mathbb{N}$

so $\underset{\sim}{k} \cdot \underset{\sim}{x} = \omega t \qquad |k \cdot x|^2 = |k^2||x|^2$

$$\rightarrow (\underset{\sim}{k} \cdot \underset{\sim}{x})(\underset{\sim}{k} \cdot \underset{\sim}{x}) = \omega^2 t^2$$

$$\rightarrow (\underset{\sim}{k} \cdot \underset{\sim}{k})(\underset{\sim}{x} \cdot \underset{\sim}{x}) = \omega^2 t^2$$

$$\rightarrow \frac{|\underset{\sim}{k}|^2}{\omega^2} = \frac{t^2}{|X|^2} \rightarrow c_p = \pm \frac{\sqrt{|k|^2}}{\omega} = \frac{\sqrt{k^2 + l^2}}{\omega} \; !$$

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